Andrew Goldberg

Math Week3 Assignment

071815

**ANSWER KEY:**

**2.34 Card game.**

**A.**

Expected winnings = %red card(winnings) + %spades(winnings) + (%clubs – ace)(winnings) + (%ace of clubs)(winnings)

= $4.13

SD = $5.44

**B.** $5.44(With a long explanation)

**2.40 Baggage fees.**

**A.**

Expected Revenue = %no bag(fees) + %1 bag(fees) + %2bag(fees)

E(X) = $15.70

SD = $23.06

**B.**

E(X) = $1884

SD = $2767.21

Assumptions: that each traveler’s baggage needs are independent of others, which is unlikely, as many people fly in groups and share baggage space

**2.42 Selling on Ebay**

**A:**

Expected net money = (price of Mario cart – price of text book) = +$38 – $110 = -$72

SD =$6.40

**B.**

E(X) for selling text book = 10% of $110 = $11

SD = $.40

**2.46 Income and gender.**

**A.** It is a continuous distribution, with non-endpoint bins of $14,999. Most Americans have incomes between 15 to 65k, with a mode of 35 to 49.9k. From <10k to 75k, it takes the relative shape of a bell curve, then increases from 75 to >99k.

**B:** 62.2%

**C:** 25.6%

**D:** 44.7% > 25.6%

**LONG FORM ANSWERS:**

2.34 Card game. Consider the following card game with a well-shuffled deck of cards. If you draw a red card, you win nothing. If you get a spade, you win $5. For any club, you win $10 plus an extra $20 for the ace of clubs.

(a) Create a probability model for the amount you win at this game. Also, find the expected

winnings for a single game and the standard deviation of the winnings.

**Expected winnings = %red card(winnings) + %spades(winnings) + (%clubs – ace)(winnings) + (%ace of clubs)(winnings)**

E = (.5(0) + .25(5) + .231(10) + .02(30)) = **$4.13**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | red card | spade | club-ace | ace clubs |  |
| xi | 0 | 5 | 10 | 30 |  |
| P(X=xi) | 0.5 | 0.25 | 0.23 | 0.02 | u= |
| xi \* P(X = xi) | 0 | 1.25 | 2.31 | 0.58 | $4.13 |
| xi - u | -4.13 | 0.87 | 5.87 | 25.87 |  |
| (xi-u)^2 | 17.10 | 0.75 | 34.40 | 669.02 | var = |
| ((xi-u)^2)\*(P(X=xi) | 8.55 | 0.19 | 7.94 | 12.87 | $29.54 |
| std = sqrt(var) | **$5.44** |  |  |  |  |

(b) What is the maximum amount you would be willing to pay to play this game? Explain.

Assuming you can play it an infinite amount of times, and have the bankroll to withstand the variance, the most I’d be willing to pay is **$4.12**, as each play would still have a positive expected value of one cent. However, depending on the speed of the shuffle and draw, my bet would probably have to be much lower to make it worth my time.

2.40 Baggage fees. An airline charges the following baggage fees: $25 for the first bag and $35 for the second. Suppose 54% of passengers have no checked luggage, 34% have one piece of checked luggage and 12% have two pieces. We suppose a negligible portion of people check more than two bags.

(a) Build a probability model, compute the average revenue per passenger, and compute the corresponding standard deviation.

**Expected Revenue = %no bag(fees) + %1 bag(fees) + %2bag(fees)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 0 bags | 1 bag | 2 bags |  |
| xi | 0 | 25 | 60 |  |
| P(X=xi) | 0.54 | 0.34 | 0.12 | u= |
| xi \* P(X = xi) | 0 | 8.5 | 7.20 | **$15.70** |
| xi - u | -4.13 | 20.87 | 55.87 |  |
| (xi-u)^2 | 17.10 | 435.36 | 3120.94 | var = |
| ((xi-u)^2)\*(P(X=xi) | 9.23 | 148.02 | 374.51 | $531.77 |
| std = sqrt(var) | **$23.06** |  |  |  |

(b) About how much revenue should the airline expect for a flight of 120 passengers? With what standard deviation? Note any assumptions you make and if you think they are justified.

Expected revenue = 120\*15.70 = **$1884**

Standard deviation = SQRT((120^2)(531.77)) = **$2767.21**

Assumptions: that each traveler’s baggage needs are independent of others, which is unlikely, as many people fly in groups and share baggage space

2.42 Selling on Ebay. Marcie has been tracking the following two items on Ebay:

• A textbook that sells for an average of $110 with a standard deviation of $4.

• Mario Kart for the Nintendo Wii, which sells for an average of $38 with a standard deviation of $5.

(a) Marcie wants to sell the video game and buy the textbook. How much net money (profits - losses) would she expect to make or spend? Also compute the standard deviation of how much she would make or spend.

Expected net money = (price of Mario cart – price of text book) = +$38 – $110 = **-$72**

SD = SQRT((1^2)(4^2) + (1^2)(5^2)) = SQRT(41) = **$6.40**

(b) Lucy is selling the textbook on Ebay for a friend, and her friend is giving her a 10% commission (Lucy keeps 10% of the revenue). How much money should she expect to make? With what standard deviation?

E(X) for selling text book = 10% of $110 = **$11**

SD = 10% of $4 = **$.40**

2.46 Income and gender. The relative frequency table below displays the distribution of

annual total personal income (in 2009 ination-adjusted dollars) for a representative sample of

96,420,486 Americans. These data come from the American Community Survey for 2005-2009.

This sample is comprised of 59% males and 41% females.

Income Total

$1 to $9,999 or loss 2.2%

$10,000 to $14,999 4.7%

$15,000 to $24,999 15.8%

$25,000 to $34,999 18.3%

$35,000 to $49,999 21.2%

$50,000 to $64,999 13.9%

$65,000 to $74,999 5.8%

$75,000 to $99,999 8.4%

$100,000 or more 9.7%

(a) Describe the distribution of total personal income.

It is a continuous distribution, with non-endpoint bins of $14,999. Most Americans have incomes between 15 to 65k, with a mode of 35 to 49.9k. From <10k to 75k, it takes the relative shape of a bell curve, then increases from 75 to >99k.

(b) What is the probability that a randomly chosen US resident makes less than $50,000 per year?

21.2+18.3+15.8 + 4.7 + 2.2 = 62.2%

(c) What is the probability that a randomly chosen US resident makes less than $50,000 per year and is female? Note any assumptions you make.

=Total under 50k \* percent women overall

=62.2% \* 41.1% = **25.6%**

Assumption: men and women are distributed equally throughout the income distribution. Which is wrong; women, on average, make less.

(d) The same data source indicates that 71.8% of females make less than $50,000 per year. Use this value to determine whether or not the assumption you made in part (c) is valid.

71.8% > 41.1%

=Total under 50k \* actual percent of women under 50k

=62.2% \* 71.8% = **44.7% > 25.6%**